Monday 9:00 - 9:50

Shape optimization for nonlocal anisotropic energies

Maria Giovanna Mora

Department of Mathematics
University of Pavia
Italy

e-mail: mariagiovanna.mora@unipv.it

Nonlocal shape optimization problems involving interaction energies with competing repulsive and attractive terms are of interest in a variety of applications and have been extensively studied in the last decades in the mathematical community. In this talk we will consider a family of nonlocal energies in 3d defined on sets with prescribed mass, where the repulsive interaction is an anisotropic variant of the Coulomb kernel and the attractive interaction is quadratic. Under the sole assumption of strict positivity of the Fourier transform of the anisotropic kernel, we will show that there is a critical value of the mass, above which ellipses are the unique minimizers and below which existence of minimizers fails. If instead the Fourier transform is just nonnegative, there is a dichotomy: either there is a critical mass as in the previous case or ellipses are minimizers for every mass. This behavior is related to the shape of minimizers when considering the energy on the larger class of measures with prescribed mass. This is a joint work with Riccardo Cristoferi (Radboud University) and Lucia Scardia (Heriot-Watt University).

Monday 9:50 - 10:40

Strong convergence of vorticity in the 2D incompressible viscosity limit

Emil Wiedemann

Department of Mathematics University of Erlangen-Nuremberg
Germany

e-mail: emil.wiedemann@fau.de

If the initial vorticity of a two-dimensional incompressible flow is in L^p, then it is classically known that solutions of the Navier-Stokes equations converge to a solution of Euler in the zero viscosity limit. Here, the convergence of the corresponding vorticities is only weak. We will present some recent results on how to upgrade to strong convergence of vorticity. The problem is particularly interesting in a bounded domain.

Monday 11:10-11:35

Linearization of finite-strain poro-visco-elasticity with degenerate mobility

Willem van Oosterhout

WIAS Berlin
Germany

e-mail: willem.vanoosterhout@wias-berlin.de

A quasistatic nonlinear model for finite-strain poro-visco-elasticity is considered in the Lagrangian frame using Kelvin-Voigt rheology. The model consists of a mechanical equation which is coupled to a diffusion equation with a degenerate mo- bility. Having shown existence of weak solutions in a previous work, the focus is first on showing boundedness of the concentration using Moser iteration. Afterwards, it is assumed that the external loading is small, and it is rigorously shown that solutions of the nonlinear, finite-strain sys-tem converge to solutions of the linear, small-strain system.

Monday 11:35-12:00

Dimension reduction of a thermo-visco-elastic problem at small strains

Moritz Gau

WIAS Berlin
Germany

e-mail: moritzimmanuel.gau@wias-berlin.de

The derivation of dimension-reduced models is a classical problem in continuum mechanics. In this talk, we consider a thermodynamically consistent nonlinear thermo-visco-elastic model at small strains for thin domains. The system of PDEs consists of the linear momentum equation (without inertia terms) for the displacement, coupled to the heat equation for the temperature. We discuss the rigorous derivation of an effective two-dimensional model for isotropic materials as the thickness of the plate tends to zero, extending results by Blanchard-Francfort 1987 and Licht 2013, who considered only linearised thermo-elasticity and visco-elasticity, respectively.

By rescaling the domain to a fixed thickness, we prove suitable a priori estimates to extract converging subsequences and pass to the limit. Since the viscous dissipation in the heat equation is only L^1-integrable, the derivation of a priori estimates utilises appropriate nonlinear tests as invented by Boccardo-Gallouët 1989. In the limit, the temperature becomes constant in the vertical direction and the displacement is of Kirchhoff–Love type, so that, in a certain sense, the limits only depend on the two-dimensional cross-section of the plate. The limiting model consists of three equations: the heat equation, and the mechanical equation splits into a membrane and a fourth-order flexural equation. Due to the viscous effects, the out-of-plane parts of the rescaled linearised strains $(\widehat{\kappa}_{ij})$ become solutions of ODEs in the limit. Especially $\widehat{\kappa}_{33}$ obtains a more complex structure compared to elastic and thermo-elastic models, and it appears as in additional coupling in all three equations.

This is a joint work with Matthias Liero (WIAS).

Monday 14:25 - 14:50

A variational perspective on auxetic metamaterials of checkerboard type

Dominik Engl

Faculty of Mathematics and Geography
Catholic University of Eichstaett-Ingolstadt
Germany

e-mail: Dominik.Engl@ku.de

Auxetic metamaterials have the counterintuitive property that they expand perpendicular to applied forces under stretching. In this talk, we discuss homogenization via Gamma-convergence for elastic materials with stiff checkerboard-type heterogeneities under the assumption of non-self-interpenetration. Our result rigorously confirms these structures as auxetic. The challenging part of the proof is determining the admissible macroscopic deformation behavior, or in other words, characterizing the weak Sobolev limits of deformation maps whose gradients are locally close to rotations on the stiff components. To this end, we establish an asymptotic rigidity result showing that, under suitable scaling assumptions, the attainable macroscopic deformations are affine conformal contractions. Our strategy is to tackle first an idealized model with full rigidity on the stiff tiles and then transfer the findings to a model with diverging elastic constants. The latter requires a new quantitative geometric rigidity estimate for non-connected touching squares and a tailored Poincaré-type inequality for checkerboard structures. This is joint work with Wolf-Patrick Düll (University of Stuttgart) and Carolin Kreisbeck (KU Eichstätt-Ingolstadt).

Monday 14:50-15:15

Equilibrium Solutions to Thermodynamically Correct Electro-Energy-Reaction-Diffusion Systems

Michael Kniely

Faculty of Mathematics
University of Graz Austria

e-mail: michael.kniely@uni-graz.at

We consider a thermodynamically consistent framework for reaction-diffusion systems mode-ling the evolution of a finite number of charged species. This approach covers, in particular, a large class of inorganic semiconductor-type models. Here, thermodynamical consistency refe-rs to charge and energy conservation and the production of entropy. This is achieved by formu-lating the model as a gradient flow system in Onsager form (for the concentrations and the in-ternal energy) coupled to Poisson's equation (for the electrostatic potential).

We will first focus on the structure of the Onsager operator and its relation to the electrostatic potential. Another aspect is the well-posedness of the corresponding equilibrium problem. By resorting to the Lagrange formalism, one can rewrite this entropy maximization problem under the constraints of charge and energy conservation as a convex minimization problem. We will see that this problem admits a unique solution, hence, a unique thermodynamic equilibrium exists. This project is joint work with Katharina Hopf and Alexander Mielke.

Monday 15:15-15:40

Standing waves at a material interface in nonlinear Maxwell equations

Tomáš Dohnal

Institute for Mathematics
Martin Luther University Halle-Wittenberg
Germany

e-mail: `tomas.dohnal@mathematik.uni-halle.de`

This is a joint work with Maximilian Hanisch and Runan He (Martin Luther University Halle-Wittenberg) as well as with Giulio Romani (Insubria Uni-versity).

We consider cubically nonlinear Maxwell equations for an interface of two generally dispersive, i.e. frequency dependent, materials, and study the existence of solutions localized at the interface and propagating in the form of a plane wave in a direction tangential to the interface.

The main physical application of this set-up is to surface plasmon polaritons (SPPs) in a metal/dielectric setting. In the time harmonic case the *linear* Maxwell equations reduce to an operator pencil problem which is non-self-adjoint in the presence of metals.

$$\nabla \times \nabla \times E - \omega^2 \epsilon(x_1, \omega) E = 0, \quad \epsilon : \mathbb{R} \times \mathbb{C} \to \mathbb{C},$$
$$E(x) = e^{ikx_2} u(x_1), k \in \mathbb{R}, u : \mathbb{R} \to \mathbb{C}^3.$$

The frequency ω plays the role of a spectral parameter. The interfa-ce is at $x_1 = 0$. Both the transverse electric case $u(x_1) = (0, 0, \phi(x_1)))^T$ and the transverse magnetic case $u(x) = (\phi_1(x_1), \phi_2(x_1), 0)^T$ are studied. First, we discuss bifurcations of nonlinear SPPs from linear ones given by simple isolated real eigenvalues of the operator pencil. Real eigenvalues occur in PT symmetric settings, where loss and gain are perfectly balanced. The bifurcation and the asymptotic expansion of the solutions are proved via a fixed point ar-gument. Numerical computations are provided to support the analysis.

In the case of a complex frequency ω we find solutions of the nonlinear time dependent Maxwell equations in the form of series of odd harmo-nics $e^{i(2m+1)\omega t}$ multiplied by spatial profiles localized at the interface. The existence problem reduces to a sequence of linear inho-mogeneous equations. In contrast to the self-adjoint case the solvability of these equations is generically satisfied.

References

[1] T. Dohnal and G. Romani. Eigenvalue Bifurcation in Doubly Nonlinear Problems with an Application to Surface Plasmon Polaritons. *Nonlinear Differ. Equ. Appl.* **28** (9) (2021)

[2] T. Dohnal and R. He. Bifurcation in Nonlinear Eigenvalue Problems Modelling Transverse Magnetic Surface Plasmon Polaritons. *J. Math. Anal. Appl.*, **538** (2), 128422 (2024)

Tuesday 9:00 - 9:50

Regularity results for a static relaxed micromorphic model

Dorothee Knees

Institute for Mathematics
University of Kassel
Germany

e-mail:
dknees@mathematik.uni-kassel.de

The relaxed micromorphic model is a generalised continuum model allowing to describe for instance size effects of microstructured solids. The state of the solid subject to external loads is characterized by the displacement field $u : \mathbb{R}^{\nvDash} \supset \Omega \to \mathbb{R}^3$ and the microdistortion tensor $P : \Omega \to \mathbb{R}^{3\times 3}$. The corresponding system of partial differential equations consist of the system of linear elastici-ty that is coupled with a system of Maxwell type for the distortion tensor P. We will discuss the regularity of weak solutions of linear and nonlinear versions of this mo-del under different assumptions on the smoothness of the domain. The main ingredients for the proofs are the Helmholtz decomposition and refined difference qoutient arguments based on generalized inner variations in combination with a Piola-type transformation. This is joint work with Patrizio Neff (Duisburg-Essen) and Sebastian Owczarek (Warsaw).

Tuesday 9:50 - 10:40

A fractional approach to strain-gradient plasticity

Stefano Almi

Department of Mathematics and Applications
University of Naples
Italy

e-mail: stefano.almi@unina.it

We discuss the derivation of a strain-gradient theory for plasticity as the Gamma-limit of discrete dislocation fractional energies, without the introduction of a core-radius. We start from a nonlocal model of semi-discrete dislocations, in which the stored elastic energy is computed via the fractional gradient of order $1 - \alpha$. In the limit as α tends to 0, we characterize the self-energy and show its consistency with previous literature.

Tuesday 11:10 - 11:35

Linearization in magnetoelasticity

Anastasia Molchanova

Institute for Analysis and Scientific Computing
TU Vienna
Austria

e-mail: anastasia.molchanova@tuwien.ac.at

In this talk, we discuss a model of nonlinear magnetoelasticity, where magnetization is defined within the Eulerian configuration while elasticity resides in the Lagrangian one. Through rigorous analysis, we rigorously derive a linearized model coinciding with the standard one by DeSimone and James (J. Mech. Phys. Solids, 50(2):283–320, 2002), where the zero-stress strain exhibits a quadratic dependence on magnetization. We establish the relationship between the nonlinear and linear models in terms of Gamma-convergence and the convergence of minimizers. This is a join work with S. Almi and M. Kružík.

Tuesday 11:35 - 12:00

Fast-slow limits for gradient flows on metric graphs

Georg Heinze

WIAS Berlin
Germany

e-mail:
georg.heinze@wias-berlin.de

We introduce a model for drift-diffusion equations on metric graphs (graphs where each edge is associated to a 1D interval). The model incorporates mass exchange between the metric edges and reservoirs located at the vertices. The resulting dynamics can be described in a framework based on the Energy-Dissipation-Principle (EDP) formulation of gradient flows. Building on this description, we explore the microscopic deri-vation of the model as well as fast-diffusion and Kirchhoff limits. To complement our analytic studies, we also conduct a numerical investigation of the limiting processes.

Joint work with J.-F. Pietschmann and A. Schlichting.

Tuesday 14:00 - 14:50

Quantitative stochastic homogenization of energies in fracture mechanics

Julian Fischer

Institute of Science and Technology
Klosterneuburg bei Wien
Austria

e-mail: Julian.Fischer@ist.ac.at

The quantitative homogenization of nonlinear random materials is by now rather well-understood in the case of uniformly convex energies. On the other hand, in the case of nonconvex problems - such as Griffith-type models in fracture mechanics - the theory is still in its infancy. In Griffith-type models for fracture, the formula for the homogenized fracture energy leads to a problem of finding surfaces of minimal energy in a random environment. We establish an algebraic rate of convergence (with respect to the length of correlations in the medium) for the energy of such minimizing surfaces in the case of statistically isotropic media. We then discuss possible implications concerning a quantitative homogenization theory for Mumford-Shah-like functionals with random coefficients.

Tuesday 14:50 - 15:15

Quantitative homogenization of forced mean curvature flow through a random field of obstacles

Jonas Ingmanns

Institute of Science and Technology
Klosterneuburg bei Wien
Austria

e-mail: Jonas.Ingmanns@ist.ac.at

We are interested in the motion of interfaces by forced mean curvature flow through a field of random obstacles. The effective large scale behavior is expected to be a first order motion. However, previous results required a global minimum speed for the interface and hence the absence of any actual obstacles.

We obtain a quantitative homogenization result even with actual obstacles, potentially allowing the interface to get stuck locally, eventually leaving behind enclosures behind a main front. So far in this regime not even a qualitative stochastic homogenization result had been available. The existence of a global minimum speed is replaced with the assumption that – for a range of small thin boxes – the interface ‘percolates’ from bottom to top at some minimum speed with high enough probability. (joint work with Julian Fischer)”

Tuesday 15:15 - 15:40

A convergence result for the horizontal mean curvature flow

Raffaele Grande

Institute of Information Theory and Automation
Czech Academy of Sciences, Prague
Czechia

e-mail: grande@utia.cas.cz

The evolution by horizontal mean curvature flow was broadly studied for its applications in applied sciences (e.g. Citti-Sarti model). It represents the contracting evolution of a hypersurface embedded in a sub-Riemannian geometry, in which is a setting where only some curves (called horizontal curves) are admissible by definition. This may lead to the existence of some points of the hypersurface in which it is not possible to define the so-called horizontal normal. These points are called characteristic. To avoid this problem, it is possible to use the notion of Riemannian approximation of a sub-Riemannian geometry applied to the horizontal mean curvature flow. In this talk there will presented some results obtained in collaboration with F.Dragoni (Cardiff University) about the relation between the solution of the horizontal mean curvature flow and the solution of Riemannian approximated mean curvature flow.

Tuesday 16:10 - 16:35

Homogenisation of local colloid evolution in- duced by reaction and diffusion

David Wiedemann

Faculty of Mathematics
TU Dortmund
Germany

e-mail: `david.wiedemann@math.tu-dortmund.de`

We consider the homogenisation of a coupled reaction-diffusion process in a porous medium with evolving microstructure. A concentration-dependent reaction rate at the interface of the pores and the solid matrix induces a concentration-dependent evolution of the domain, which makes the evolution fully coupled with the reaction-diffusion process. In order to pass to the homogenisation limit, we employ the two-scale-transformation method. Thus, we homogenise the highly non-linear problem in a periodic and in-time cylindrical domain instead. The homogenisation result is a reaction-diffusion equation, which is coupled with an internal variable, representing the local evolution of the pore structure.

This is joint work with M. Peter (Augsburg).

Tuesday 16:35 - 17:00

Inertial Balanced Viscosity (IBV) solutions to rate-independent systems.

Filippo Riva

Department of Decision Sciences
Bocconi University, Milan
Italy

e-mail: `filippo.riva@unibocconi.it`

Rate-independent evolutions frequently occur in mechanics when the problem under consideration presents such small rate-dependent effects, as inertia or viscosity, that can be neglected. If the driving potential energy is nonconvex any (reasonable) solution necessarily exhibits time-discontinuities. To describe the behaviour at jumps, in the last decades the variational notions of Energetic and Balanced Viscosity (BV) solutions have been developed, the latter as a refinement of the former. While for Energetic solutions the cost at jumps is described only in terms of the rate-independent dissipation potential, in the case of Balanced Viscosity solutions it arises from a suitable coupling (based on the De Giorgi's approach to gradient flows) between this dissipation potential and a smaller and smaller viscosity potential, which is reminiscent of an original presence of viscous damping. In this talk, after an overview on Energetic and BV solutions, we present an enhanced notion of solution to rate-independent systems, named Inertial Balanced Viscosity (IBV) solution, which in addition takes into account small inertial effects dictated by Newton's law. We show how IBV solutions can be obtained as a vanishing inertia and viscosity limit of second order dynamic evolutions, and we present some applications in mechanics. We also present a multiscale Minimizing Movements algorithm which can be used to build such solutions. This is based on joint works with P. Gidoni, G. Scilla and F. Solombrino.

Wednesday 9:00 - 9:50

Variational methods for dynamical PDEs

Malte Kampschulte

Faculty of Mathematics and Physics
Charles University, Prague
Czechia

e-mail: kampschulte@karlin.mff.cuni.cz

Energy-functionals are a common tool in the study of PDEs, as they provide natural estimates. But for physical PDEs they can also be used as the starting point for modelling. Instead of deriving the PDE mainly as sum of kinematics and force balances, the role of the latter can equivalently be replaced by the functionals responsible for these forces. Viewing the problem from that angle then allows the use of mainly variational methods to naturally generate solutions without need for comparatively unphysical approximations, as the system will always be consistent from the start. The aim of this talk is to present an overview of newer and older results showing both approaches to modelling, as well as proving existence for various problems from different parts of continuum mechanics. This is based in part on joint works with B.Benešová, D.Breit, A.Češík, G.Gravina and S.Schwarzacher.

Wednesday 9:50 - 10:40

Li-Yau and Harnack estimates for nonlocal diffusion problems

Rico Zacher

Institute of Applied Analysis
University of Ulm
Germany

e-mail: rico.zacher@uni-ulm.de

I will present recent results on differential Harnack inequalities of Li-Yau type for a large class of nonlocal diffusion equations. This includes problems on infinite discrete structures (graphs) on which arbitrarily long jumps are possible, problems in Euclidean space with a fractional Laplace operator and hybrid problems in a discrete-continuous setting which cover certain systems of reaction-diffusion equations. One of the main difficulties is that the classical chain rule is not valid for the nonlocal operators under consideration. Additionally, if one wants to adopt Li and Yau's approach from their famous 1986 paper (Acta. Math.), new curvature-dimension (CD) inequalities are required, since the classical Bakry-Emery condition based on the Gamma calculus is no longer suitable. This also touches on the fundamental question of how to define lower curvature bounds on discrete structures in a meaningful way. In addition to the approach using CD inequalities, I will present another method which is based on heat kernel representations of the solutions and consists in reducing the problem to the heat kernel. I will also show that the new nonlocal Li-Yau type estimates are strong enough to derive Harnack inequalities. This is partly joint work with D. Dier, M. Kassmann, S. Kräss, A. Spener and F. Weber.

Wednesday 11:10 - 11:35

Measure-valued solutions for non-associative finite plasticity

Andreas Vikelis

Faculty of Civil Engineering
CTU Prague
Czechia

e-mail: `andreas.panagiotis.vikelis@fsv.cvut.cz`

The variational treatment of evolutionary nonassociative elasto-plasticity at finite strains remains unexplored. In this direction, following the concept of energetic solutions, we present an existence result for measure-valued so- lutions of the quasistatic evolution problem which are stable and balance the energy. In particular, we apply a modification of the standard time- discretization scheme, considering Young measures generated by piecewise constant interpolants of time-discrete solutions of a properly defined mini- mization problem. A key point in our analysis is the limit passage in the dissipation. The later calls for time-continuity properties of the stresses which are not expected in the quasistatic framework. To overcome this obstacle we introduce a regularization of the generalized stress in the definition of our energetic solutions. Joint work with Ulisse Stefanelli.

Weighted Energy-Dissipation approach to semilinear gradient flows with state-dependent dissipation

Riccardo Voso

Faculty of Mathematics
University of Vienna
Austria

e-mail: riccardo.voso@univie.ac.at

We investigate the Weighted Energy-Dissipation variational approach to semilinear gradient flows with state-dependent dissipation. A family of parameter-dependent functionals defined over entire trajectories is introduced and proved to admit global minimizers. These global minimizers correspond to solutions of elliptic-in-time regularizations of the limiting causal problem. By passing to the limit in the parameter we show that such global minimizers converge, up to subsequences, to a solution of the gradient flow.

Wednesday 14:00 - 14:50

Gradient flow formulations for jump processes in metric spaces

Riccarda Rossi

Department of Mechanical and Industrial Engineering
University of Brescia
Italy

e-mail:
riccarda.rossi@unibs.it

We will discuss the variational formulation of Markov jump processes in metric spaces, as generalized gradient flows. After settling the theory in the case of bounded kernels, we will move to singular kernels. We will obtain well-posedness results if the densities of the laws with respect to a reversible invariant measure are bounded from above and above, and a certain Lipschitz density result holds.

Joint work with Jasper Hoeksema and Oliver Tse (TUE Eindhoven).

Wednesday 14:50 - 15:15

On different constitutive relations and boundary conditions for fluids

Erika Kokavcová

Institute of Science and Technology
Klosterneuburg bei Wien
Austria

e-mail:
erika.kokavcova@ist.ac.at

In this talk we will review our work on implicitly constituted fluids with M. Bulíček and J. Málek. We will also present some recent results in numerical analysis for different boundary conditions obtained with A. Gazca Orozco, F. Gmeineder and T. Tscherpel.

Wednesday 15:15 - 15:40

Dynamics of wedge disclinations

Marco Morandotti

Department of Mathematical Sciences
Politecnico di Torino
Italy

e-mail:
marco.morandotti@polito.it

Disclinations in crystalline materials are point defects that are responsible for rotational kinematic incompa-tibility [4]. They are characterised by the so-called Frank angle, measuring the severity of the lattice mis-match. The variational setting and semi-discrete modeling of a systems of disclinations has been developed in [3] resorting to the Airy stress function. In this talk, we present recent results on the dynamics of disclinations in a two-dimensional domain. Disclinations move by energy minimization, in a similar fashion as dislocations do [1,2]. We study the well-posedness of the ODE governing the motion of a system of disclinations, with particular attention to the sim-ple, yet illuminating cases of one disclination alone or two disclinations in the domain. An analysis of collision times is performed, and we also show how to account for the possible presence of preferred directions of motion determined by the crystalline structure. Finally, we show numerical evidence of this dynamics. This is work in collaboration with Pierluigi Cesana (Kyushu University) and Alfio Grillo and Andrea Pastore (Politecnico di Torino).

[1] Blass, Fonseca, Leoni, Morandotti, SIAM J. Appl. Math. 2015.

[2] Cermelli, Gurtin, Arch. Ration. Mech. Anal. 1999.

[3] Cesana, De Luca, Morandotti, SIAM J. Math. Anal. 2024.

[4] Volterra, Annales scientifiques de l'École Normale Supérieure, 1907.

Thursday 9:50 - 10:40

Measure-valued structured deformations

Elvira Zappale

Dipartimento di Scienze di Base e Applicate per l'Ingegneria
University of Rome, Sapienza
Italy

e-mail: elvira.zappale@uniroma1.it

I will present the theory of Structured Deformations (introduced by Del Piero and Owen) which allows for a unified theory of deformations of continua. In particular I will introduce the energy associated with a structured deformation, also in the measure valued case. It is de-fined via relaxation departing from energies associated with classical deformations. Several integral representation results for the energy functional (according to different settings) will be provided both in the unconstrained case and under Dirichlet boundary conditions. The talk is mainly based on results obtained in collaboration with Stefan Krömer, Martin Kružík and Marco Morandotti and with Ana Cristina Barroso and José Matias.

Thursday 11:10 - 12:00

Convexity and uniqueness in the Calculus of Variations

Bernd Kirchheim

Mathematical Institute
University of Leipzig
Germany

e-mail: kirchheim@math.uni-leipzig.de

Whereas general existence results for minimizers of (vectorial) variational problems are clearly related to (coercivity) and Morrey's quasiconvexity, the situation becomes much more constrained if also uniqueness of the minimizers is required for all linear perturbation of the energy. In this case a rather natural notion of functional convexity arises in a general Banach space context. We will discuss what are the specific implications for energy densities of integral cost functions.

This is joint work with J. Campos Cordero (Mexico), J. Kollar (Prag) and J.Kristensen (Oxford).